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TEMPERATURE AND MOISTURE DISTRIBUTIONS INSIDE A LAYER OF MOIST MATERIAL PLACED ON A HOT ISOTHERMAL SURFACE

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The problem of contact drying of a layer of moist material placed on a hot isothermal surface is considered. A system of heat and mass transfer equations and equations for the initial and boundary conditions are proposed. Luikov's theoretical approach is utilized, but all relevant thermophysical coefficients are considered to be fully dependent on the temperature and moisture content inside the material during the process. The model is solved numerically. Results for temperature and moisture distributions inside the layer of the material obtained using this model are compared to results obtained by solving a model with constant thermophysical coefficients in the governing equations, formed by using the same type of heat and mass transfer equations and equations for the initial and boundary conditions. The process is also examined experimentally, which provided results that enable verification of the proposed models.

Introduction. Analytical studies of heat and mass transfer during drying processes with intense heat supply rarely succeed in developing mathematical models accurate enough for practical use. Considering the great complexity of the mechanisms of heat and mass transfer during drying and the difficulties arising from the usual asymmetry of the boundary conditions in processes with nonconvective heat supply, the investigations mentioned introduce some inadmissible assumptions and simplifications of the initial systems of equations in order to make their analytical solution possible. These assumptions degrade the physical conformity of the model and lead to incorrect solutions of drying dynamics. The commonly used postulate that the values of the thermophysical and transport properties of the dried material are unconditionally constant during the process is especially deleterious.

The importance of taking into account all thermophysical and transport properties as fully dependent on the temperature and moisture content inside the material during the process will be shown on the example of a simple contact drying process. A mathematical model that consists of a system of heat and mass transfer equations and equations for the initial and boundary conditions at the contact and free surface that are based on Luikov's theoretical approach [1, 2, 3] are presented, but all thermophysical coefficients are considered to be fully dependent on the temperature and moisture content inside the material during the process. Solving such a complicated system of equations would be impossible without utilizing adequate numerical procedures.

Formulation of the Model. The one-dimensional case of a layer of moist material placed on a hot plate of constant temperature, with a convective air stream of certain parameters being blown over the opposite free surface of the sample, is considered.

Since the observed temperatures of the heating surface and the resulting temperatures inside the sample are well under the level that causes boiling of liquid moisture inside the material, the influence of the total pressure gradient on the heat and mass transfer could be neglected [4, 6, 7, 9]. Consequently the local

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values of temperature and moisture content inside the material could be considered the relevant potentials for heat and mass transfer during the process.

Assuming that all thermophysical coefficients are dependent on the temperature and moisture content of the dried material, and that the shrinkage of the material during drying can be neglected, the resulting system of dimensionless equations for heat and mass transfer inside the layer of material is

$$S_c\,(T,\,U)\,\frac{\partial T}{\partial \mathrm{Fo}} = \frac{\partial}{\partial X}\left(S_\lambda\,(T,\,U)\,\frac{\partial T}{\partial X}\right) - \mathrm{Ko}\,S_r\,(T,\,U)\,S_\varepsilon\,(T,\,U)\,\frac{\partial U}{\partial \mathrm{Fo}}\,, \tag{1}$$

$$\frac{\partial U}{\partial \mathrm{Fo}} = \mathrm{Lu}\,\frac{\partial}{\partial X}\left(S_{a_{\mathrm{moist}}}\,(T,\,U)\,\frac{\partial U}{\partial X}\right) - \mathrm{Lu}\,\mathrm{Pn}\,\frac{\partial}{\partial X}\left(S_{a_{\mathrm{moist}}}\,(T,\,U)\,S_\delta\,(T,\,U)\,\frac{\partial T}{\partial X}\right). \tag{2}$$

Equations (1) and (2) are subjected to the following initial conditions:

$$T = 0\,, \tag{3}$$

$$U = 0\,. \tag{4}$$

The convective boundary conditions for heat and water vapor transfer at the free surface of the material (X = 1) are

$$\left(S_\lambda\,(T,\,U)\,\frac{\partial T}{\partial X}\right)_{X=1} + \mathrm{Ki}_q\,(\mathrm{Fo}) + \mathrm{Lu}\,\mathrm{Ko}\left[S_r\,(T,\,U)\,(1 - S_\varepsilon\,(T,\,U))\right]_{X=1}\mathrm{Ki}_{\mathrm{m}}\,(\mathrm{Fo}) = 0\,, \tag{5}$$

$$-\left(S_{a_{\mathrm{moist}}}\,(T,\,U)\,\frac{\partial U}{\partial X}\right)_{X=1} + \mathrm{Pn}\left(S_{a_{\mathrm{moist}}}\,(T,\,U)\,S_\delta\,(T,\,U)\,\frac{\partial T}{\partial X}\right)_{X=1} + \mathrm{Ki}_{\mathrm{m}}\,(\mathrm{Fo}) = 0\,. \tag{6}$$

The boundary condition for heat transfer at the contact surface is obtained over the known temperature of the heating surface.

If the contact resistance to heat transfer in the zone of junction of these two surfaces can be neglected, the boundary condition is

$$T_{X=0} = \frac{t_{\mathrm{h}} - t_0}{t_0}\,. \tag{7a}$$

However, in most of the cases the value of the contact resistance to heat transfer cannot be disregarded [9, 10]. Analytical characterization of this contact resistance for different periods of the process and determination of its exact value are hardly possible in view of the complexity of the heat transfer processes at the contact of the heating surface and the deformable moist surface of the dried material. For these reasons the contact resistance is introduced through an empirical parameter $\Delta t(\tau)$ that gives the value of the temperature drop of the contact surface due to the contact resistance during the process:

$$T_{X=0} = \frac{t_{\mathrm{h}} - \Delta t\,(\tau) - t_0}{t_0}\,. \tag{7b}$$

The boundary condition for mass transfer at the contact surface is derived assuming that there is no moisture transfer from the material to the heating surface through the contact surface:

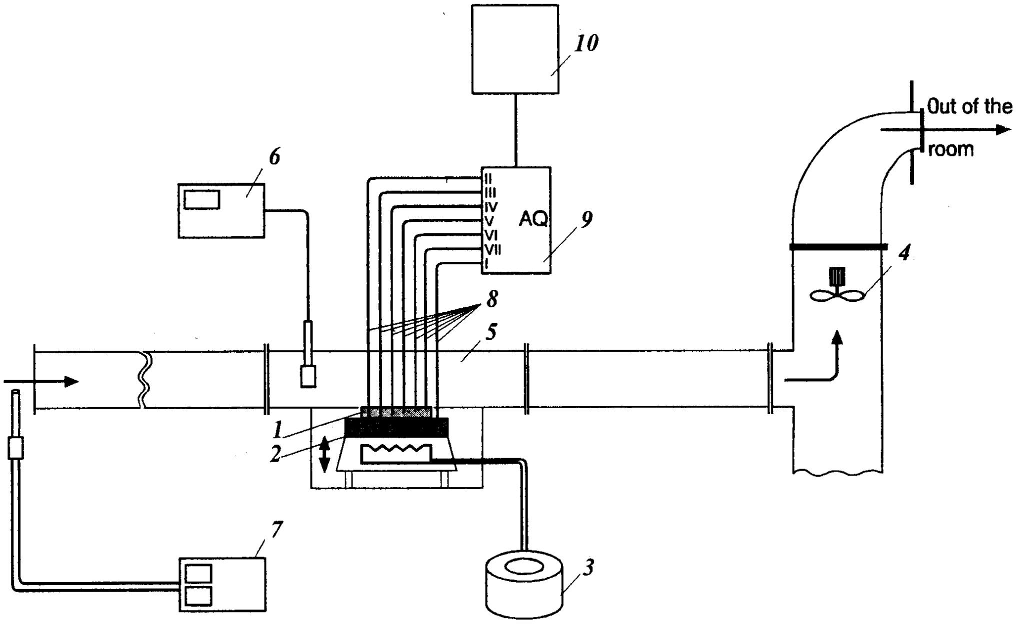


Fig. 1. Scheme of the experimental facility.

$$\left(\frac{\partial U}{\partial X}\right)_{X=0} - \mathrm{Pn}\left(S_{\delta}(T, U)\frac{\partial T}{\partial X}\right)_{X=0} = 0 . \tag{8}$$

The proposed model (1)-(8) with variable coefficients can easily be turned into a model with constant coefficients by simply introducing constant thermophysical coefficients ($S_c(T, U) = S_{\lambda}(T, U) = S_r(T, U) = S_{a_{\mathrm{moist}}}(T, U) = S_{\delta}(T, U) = 1$ and $S_{\varepsilon}(T, U) = \varepsilon = \mathrm{const}$) in the governing equations.

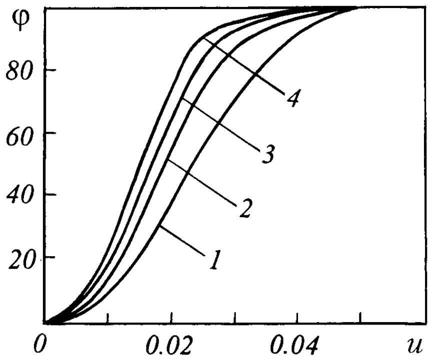
Experimental Investigation. The experimental part of the investigation is performed on the laboratory pilot facility schematically displayed in Fig. 1.

A sample of moist material having dimensions $L \times 80 \times 80$ mm (1) is placed on an electrically heated thick copper plate (2) with surface temperature regulation (3). Over the opposite, free surface of the sample air of a certain temperature and relative humidity is blown by means of an axial fan (4). Depending on the sample thickness, the relative height of the heated plate with the sample is regulated in order to level the free surface of the sample with the bottom of the air channel (5). The geometry of the air channel (the length of the air channel before the sample is 1500 mm and its cross section is 250 × 100 mm) provides a stable velocity profile over the sample. The air velocity over the sample is measured by an anemometer (6). All experiments presented are performed with an average air velocity of 2 m/sec.

The inlet temperature and relative humidity of the air stream during the experiments are measured by a Rotronic sonde (7) and maintained at values of 20°C for the temperature and 50% for the relative humidity.

For different values of the sample thickness ($L = 10$, 15, or 20 mm), the initial moisture content u_0 ($u_0 = 0.175$, 0.200, or 0.233 kg/kg), and the hot-surface temperature ($t_{\mathrm{h}} = 50$, 65, or 80°C) and for various durations of the drying process, temperature profiles and the average moisture content inside the sample are recorded.

The temperatures inside the sample during the process are measured by means of fine-wire (0.1 mm in diameter) Cr–Ni thermocouples (8), connected to the system for data acquisition (9) and processing (10). Thermocouples are inserted into the sample horizontally, following the isothermal surfaces. Inside the sample four thermocouples are positioned equidistantly. Additionally, two thermocouples are placed at the boundary surfaces of the sample, and one thermocouple is placed just under the surface of the copper plate. The temperatures of all seven thermocouples are recorded with the sample period of 10 sec.



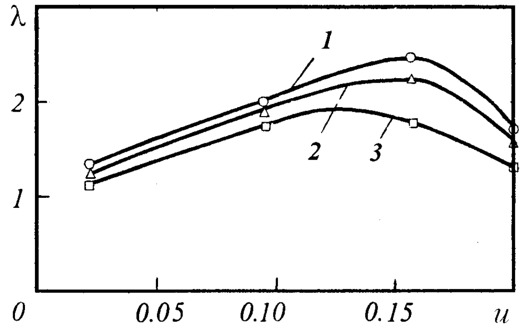


Fig. 2. Isotherms of desorption of the model material [5]: 1) $t = 20^{\circ}$C; 2) 40; 3) 60; 4) 80. φ, %; u, kg/kg.

Fig. 3. Thermal conductivity of the model material [5]: 1) $t = 20^{\circ}$C; 2) 40; 3) 80. λ, W/(m·K).

The average moisture content of the sample after a certain drying time is determined by measuring the mass of the sample before and after dehydration and the mass of the dry base of the sample after complete dehydration in a furnace.

Model Material. The material used in the experimental part of the investigation is a specified mixture of quartz sand and bentonite. The mechanical properties of this material and its ability to maintain a constant volume during drying, as well as the fact that earlier experimental investigations [4, 5, 8] determined its equilibrium states (Fig. 2) and all its relevant thermophysical and transport coefficients as functions of temperature and moisture content (Figs. 3 and 4; Eqs. (9)–(13)), make a quartz sand–bentonite mixture attractive for a wide range of experimental use.

The remaining relevant thermophysical and transport coefficients are determined as:

specific heat capacity [5]:

$$c = 612 + 0.00193 T_{\text{K}}^2 + 4178u \,, \tag{9}$$

thermal diffusivity [4]:

$$a = \frac{1 - 0.00198t}{1418 + 847.5u} \, \frac{1 - 718.3u^3 + 152.5u^2 + 0.45u}{755.8 + 0.00193t^2 + 1.054t + 4178u} \,, \tag{10}$$

thermal-gradient coefficient [5]:

$$\delta = 0.015 \exp\left[-455\,(u - 0.1)^2\right] , \tag{11}$$

phase-change criterion [4]:

$$\varepsilon = 0.9\text{H}\,(u) - 0.6\text{H}\,(u - 0.098) \,, \tag{12}$$

specific heat of evaporation [5]:

$$r = 2502 \cdot 10^3 - 2470\, t \,. \tag{13}$$

When the numerical values of temperature expressed in K for Eq. (9) and in $^{\circ}$C for Eqs. (10) and (13) and moisture content expressed in kg/kg are introduced in Eqs. (9)–(13) we obtain the values of the thermophysical properties with the dimensions given in Notation.

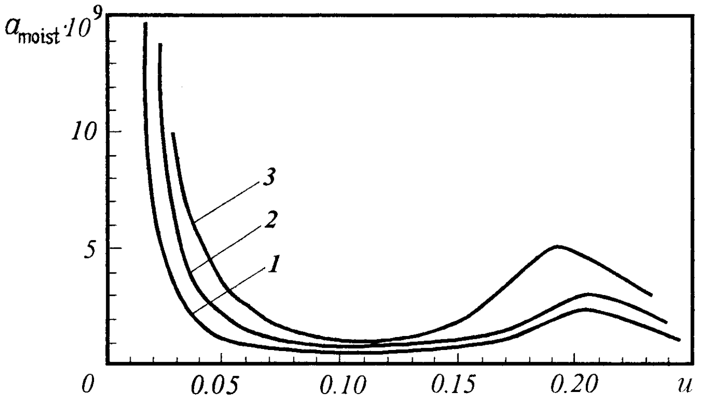


Fig. 4. Moisture diffusivity of the model material [5]: 1) t = 30°C; 2) 45; 3) 60; a_{moist}, m^2/sec.

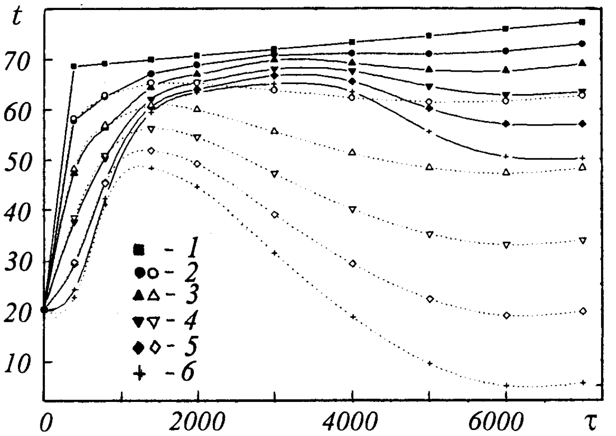


Fig. 5. Temperature curves predicted by the model with constant (dotted lines) and variable (solid lines) coefficients: 1) x = 0 mm; 2) 3; 3) 6; 4) 9; 5) 12; 6) 15; t, °C; τ, sec.

Results. Both variants of the model (with variable and constant coefficients) are solved numerically using a nonlinear finite-difference explicit numerical scheme. The model with variable coefficients is solved using the functional dependences of the relevant thermophysical and transport coefficients on the temperature and moisture content described in the previous section. The values of the thermophysical properties used in solving the model with constant coefficients are

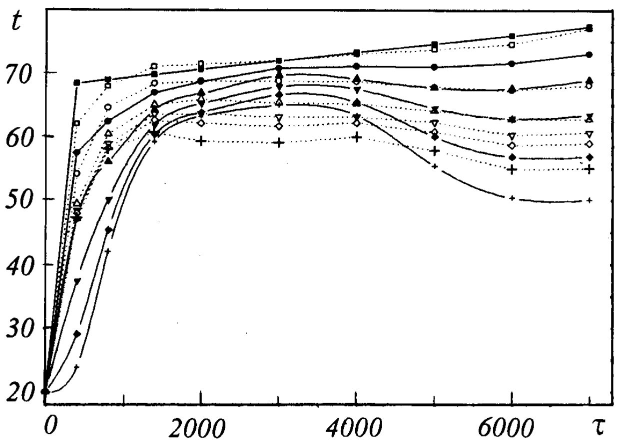
$$\lambda = 1.663 \ \text{W/(m·K)} \ ; \quad c = 1613.3 \ \text{J/(kg·K)} \ ; \quad a = 5.413 \cdot 10^{-7} \ \text{m}^2\text{/sec} \ ;$$

$$a_{\text{moist}} = 1.601 \cdot 10^{-9} \ \text{m}^2\text{/sec} \ ; \quad \delta = 1.585 \cdot 10^{-4} \ \text{kg/(kg·K)} \ ; \quad \varepsilon = 0.3 \ ; \quad r = 2{,}452{,}600 \ \text{J/kg} \ .$$

The values of the Kirpichev heat and mass flux numbers during the process, which enter Eqs. (5) and (6), are determined experimentally. These procedures are described in detail in [4].

The temperature and moisture distributions inside the layer of material obtained using the model with variable coefficients are compared to the corresponding results obtained by solving the model with constant thermophysical coefficients in the governing equations, as well as to results acquired experimentally for the same values of the process parameters.

The results shown in Figs. 5–9 are derived for the following process conditions: temperature of the heating surface 80°C; temperature of the air stream 20°C; velocity of the air stream over the free surface of



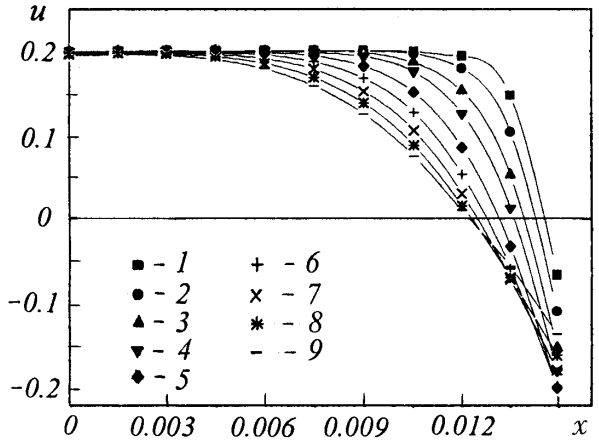


Fig. 6. Temperature curves obtained experimentally (dotted lines) and predicted by the model with variable coefficients (solid lines). For notation see Fig. 5.

Fig. 7. Moisture-content profiles calculated using the model with constant coefficients: 1) $\tau = 400$ sec; 2) 800; 3) 1400; 4) 2000; 5) 3000; 6) 4000; 7) 5000; 8) 6000; 9) 7000. x, m.

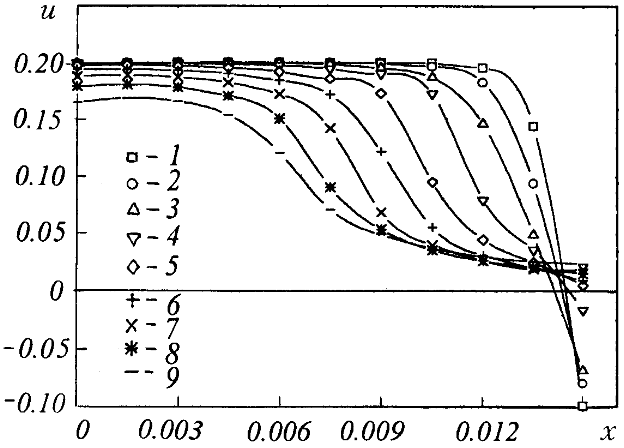
the sample 2.0 m/sec; relative humidity of the air stream 50%, thickness of the sample 15 mm; initial temperature of the material 20°C; initial moisture content of the material 0.2 kg/kg.

The temperatures acquired by numerically solving the models with constant and variable parameters for the adopted process conditions are shown on Fig. 5. The discrepancies between the temperatures predicted by these two models are significant for all the periods of drying except the initial period. During the initial period of drying the temperatures calculated by these two models are similar, because the values of the variable and constant thermophysical coefficients in the initial period of the process are still comparable. The temperatures obtained by the model with constant coefficients for the zones of the material near the free surface for longer drying times are especially inferior and without physical meaning (Fig. 5).

When compared with the experimental results (Fig. 6), the temperature profiles obtained by the variable-coefficients model show good agreement. Some discrepancies, especially during the warming-up period, are mainly a consequence of adoption of a linear time dependence for the empirical parameter of the contact resistance temperature drop $\Delta t(\tau)$ in the boundary condition for heat transfer at the contact surface (7b) during the numerical calculation. The adopted linear dependence $\Delta t(\tau)$, taken to simplify the numerical procedure, is accurate enough for all the periods of drying except the initial one, where the temperature difference between the heating and the contact surface is larger than assumed. So for calculations of temperature distributions during the warming-up period a more exact function $\Delta t(\tau)$ should be used. Of course the accuracy of the predicted temperature fields is also affected by the precision of the experimental determination of the heat and mass fluxes at the free surface of the layer.

The use of the variable-coefficients model is especially important for calculations of the moisture content of the material during drying.

As could be expected on the basis of earlier results of the constant-coefficients approach [1, 3], the moisture content profiles obtained by the model with constant coefficients (Fig. 7) have a parabolic shape. According to that model, lowering of the moisture content of the sample is effected by decreasing the curve of the parabolic moisture-content profile and withdrawing it from the free surface. Consequently, the calculation gives physically incorrect – negative – values of the moisture content for the zones of the material near the free surface. Additionally, that model practically does not show the decline of the moisture content in the zones close to the contact surface during the process, which evidently exists.



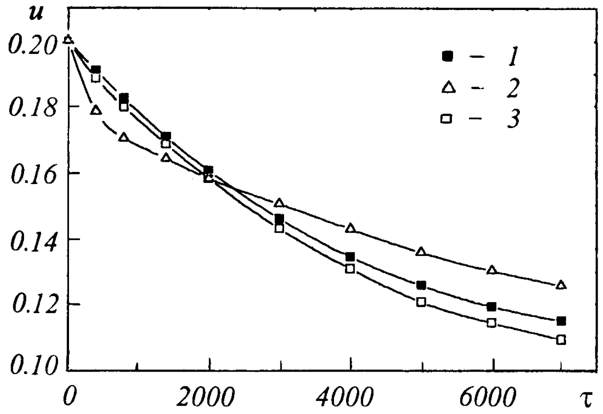


Fig. 8. Moisture-content profles calculated using the model with variable coefficients: 1) τ = 400 sec; 2) 800; 3) 1400; 4) 2000; 5) 3000; 6) 4000; 7) 5000; 8) 6000; 9) 7000.

Fig. 9. Average moisture content of the sample during the process: 1) experiment; 2) model with constant coefficients; 3) model with variable coefficients.

The moisture content profiles predicted by the model that respects all thermophysical coefficients as fully dependent on the temperature and moisture content inside the material during the process (Fig. 8) have a different shape. There are parabolic only in the initial times – during the warming-up period of the material. Only in these initial phases of the process might incorrect – negative – values of the moisture content at the free surface be obtained. During the rest of the process the moisture-content profiles, in all zones of the sample, have a physically very satisfactory shape. Besides, they show a decline of the moisture content in the zones near the contact surface that increases during the process.

In the experimental part of the investigation the data on the average moisture-content change during the process are acquired. For the utilized set of process conditions, very good agreement of the average moisture-content decrease during the process predicted by the model with variable coefficients with experimental results is obtained, while the conformity of the results predicted by the model with constant coefficients with experimental results is lesser (Fig. 9).

Conclusions. For the observed contact drying process, the temperature and moisture content profiles inside the sample predicted by the model with variable coefficients have much more physical meaning and show much better agreement with the experimental data than the corresponding results of the analogous model with constant coefficients. The proposed variable-coefficients approach is especially superior for zones of the sample closer to the free surface and for longer drying times, where the model with constant coefficients gives radically incorrect results. It might be concluded that the classical constant-coefficients approach shows inadequacy if used for modeling the heat and mass transfer during intensive drying processes and that in these cases models that respect the dependences of the thermophysical and transport properties of the dried material on its temperature and moisture content should be used.

The main obstacle to wider application of a variable-coefficient model is the need for accurate experimental determination of the dependences of several thermophysical coefficients on the temperature and moisture content for the particular dried material. Additionally, if the temperature inside dried material during the process exceeds the boiling point of the moisture contained in the material, a total pressure gradient arises and the system of partial differential equations should be completed with an equation for pressure transfer (in which additional transfer coefficients figure). In that case all the values of the thermophysical and transport properties of the material should be determined with respect to their dependences on three transfer potentials: temperature, moisture content, and pressure.

NOTATION

a_{moist}, moisture diffusivity, m^2/sec; c, heat capacity, J/(kg·K); j_{m}, mass flux and overall drying rate, (kg/(m^{2}sec)); j_q, heat flux, W/m^2; L, thickness of the sample, m; r, specific heat of evaporation, J/kg; t, temperature, $^{\text{o}}$C; t_0, initial temperature, $^{\text{o}}$C; t_{h}, temperature of the heating surface, $^{\text{o}}$C; u, moisture content, kg/kg; u_0, initial moisture content, kg/kg; x, distance from the heating surface, m; $\Delta t(\tau)$, empirical parameter for the contact resistance temperature drop, $^{\text{o}}$C; δ, thermal-gradient coefficient, kg/(kg·K); ρ_0, density of the dry material, kg/m^3; τ, drying time, sec; ε, phase-change criterion; λ, thermal conductivity, W/(m·K); H(u), Heaviside function; $X = \frac{x}{L}$, dimensionless distance from the heating surface; $T = \frac{t - t_0}{t_0}$, dimensionless temperature; $U = \frac{u_0 - u}{u_0}$, dimensionless moisture content; $S_c(T, U) = \frac{c(t, u)}{c_0}$, dimensionless heat capacity; $S_\lambda(T, U) = \frac{\lambda(t, u)}{\lambda_0}$, dimensionless thermal conductivity; $S_r(T, U) = \frac{r(t, u)}{r_0}$, dimensionless specific heat of evaporation; $S_{a_{\text{moist}}}(T, U) = \frac{a_{\text{moist}}(t, u)}{a_{\text{moist},0}}$, dimensionless moisture diffusivity; $S_\delta(T, U) = \frac{\delta(t, u)}{\delta_0}$, dimensionless thermal-gradient coefficient; $S_\varepsilon(T, U) = \varepsilon(t, u)$, dimensionless phase-change criterion; $\text{Fo} = \frac{a_0\tau}{L^2}$, Fourier number; $\text{Lu} = \frac{a_{\text{moist},0}}{a_0}$, Luikov number; $\text{Pn} = \frac{\delta_0 t_0}{u_0}$, Posnov number; $\text{Ko} = \frac{r_0 u_0}{c_0 t_0}$, Kossovich number; $\text{Ki}_q(\tau) = \frac{L j_q(L, \tau)}{\lambda_0 t_0}$, Kirpichev heat-flux number; $\text{Ki}_{\text{m}}(\tau) = \frac{L j_{\text{m}}(L, \tau)}{a_{\text{moist},0}\rho_0 u_0}$, Kirpichev mass-flux number.

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